

**AMENDMENTS TO THE CLAIMS:**

This listing of claims will replace all prior versions, and listings, of claims in the application:

**Listing of Claims:**

1-47. (Cancelled).

48. (New) A method for using a conformal structure of a surface of a multidimensional object for shape analysis manipulation, the method comprising:

(i) determining the conformal structure of the surface represented by a mesh  $M$  for performing shape analysis manipulation of the object; and

(ii) using the conformal structure to conformally map the surface to a canonical parameter domain.

49. (New) The method of claim 48, wherein the step of determining the conformal structure further includes transforming the surface into a closed surface.

50. (New) The method of claim 49, further comprising doubling the mesh  $M$  to form  $\bar{M}$ .

51. (New) The method of claim 50, wherein the doubling step is a topological doubling of the mesh  $M$  to form the doubled mesh  $\bar{M}$ .

52. (New) The method of claim 51, wherein the step of doubling the mesh  $M$  to form the doubled mesh  $\bar{M}$  further comprises:

gluing the mesh  $M$  to a second mesh  $-M$  reversely oriented from mesh  $M$  along the corresponding boundaries thereof.

53. (New) The method of claim 52, wherein the step of doubling the mesh  $M$  to form the doubled mesh  $\bar{M}$  further comprises:

deriving the second reversely oriented mesh  $-M$  from the mesh  $M$ ;

finding for each boundary vertex  $u$  on the boundary of the mesh  $M$ , represented as  $\partial M$ , such that  $u \in \partial M$ , a unique corresponding boundary vertex  $-u$  on the boundary of  $-M$ , represented as  $\partial -M$ , such that  $-u \in \partial -M$ ; and

finding for any edge  $e$  on the boundary  $\partial M$ , such that  $e \in \partial M$ , a unique boundary edge  $-e$  on the boundary  $\partial -M$ , such that  $-e \in \partial -M$ ,

wherein the gluing step further comprises deriving the doubled mesh  $\bar{M}$  from  $M$  and  $-M$  such that the corresponding vertices and edges thereof are identical.

54. (New) The method of claim 48, wherein the surface is an orientable surface.

55. (New) The method of claim 54, wherein the step of determining the conformal structure further includes generating a fundamental domain  $D_M$  of the surface.

56. (New) The method of claim 55, wherein the boundary of the fundamental domain  $D_M$ , represented as  $\partial D_M$ , comprises a cyclically ordered list of half edges, and wherein a cut graph  $G$  is comprised of a plurality of non-oriented edges attached to the half edges.

57. (New) The method of claim 55, wherein the fundamental domain is a topological disk covering the surface once.

58. (New) The method of claim 57, wherein the topological disk is a surface having a single boundary and a genus of zero.

59. (New) The method of claim 55, wherein the step of generating the fundamental domain  $D_M$  comprises the steps of:

(i) selecting and removing a first triangular face of the mesh  $M$  and placing the first triangular face in a second mesh  $M'$  comprising the fundamental domain  $D_M$ ;

(ii) selecting and removing a next triangular face adjacent to the boundary of the remaining portion of the mesh  $M$  and attaching the next triangular face to the boundary of the second mesh  $M'$  along only one edge thereof; and

(iii) repeating step (ii) until all the triangular faces have been selected and removed from the mesh  $M$  and added to the second mesh  $M'$  comprising the fundamental domain.

60. (New) The method of claim 55, wherein the step of generating the fundamental domain  $D_M$  comprises the steps of:

selecting an arbitrary face  $f_0$  on the surface of mesh  $M$  [ $f_0 \in M$ ];

setting the fundamental domain  $D_M$  to the arbitrary face  $f_0$ ;

setting the boundary of the fundamental domain  $\partial D_M$  to the boundary of the arbitrary face  $\partial f_0$ , wherein  $\partial f_0$  is defined as a sequence of half edges of  $f_0$ ;

placing all neighboring faces of arbitrary face  $f_0$  that share an edge with arbitrary face  $f_0$  in a queue  $Q$ ;

performing the steps of:

while  $Q$  is not empty:

reading the first face in  $Q$ , represented as  $f$ ;

finding the first half edge  $e_i$ , where  $i$  is an integer, of the boundary of  $f$ , referred to as  $\partial f$ , such that the opposite of  $e_i$ , referred to as  $-e_i$ , is in the boundary of the fundamental domain  $\partial D_M$ ;

replacing  $-e_i$  in  $\partial D_M$  with the next two half edges of  $\partial f$ , referred to as  $e_{(i+1)}$ ,  $e_{(i+2)}$

setting  $D_M$  to the union of  $D_M$  and  $f$ ;

putting in  $Q$  all the neighboring faces that share an edge with  $f$  in the mesh  $M$  and that are not in  $D_M$  or  $Q$ ; and

removing all adjacent half edges that are opposite to each other in  $\partial D_M$ .

61. (New) The method of claim 48, wherein the step of determining the conformal structure comprises determining a homology basis for the surface by determining the independent loops of the cut graph of the surface.

62. (New) The method of claim 61, wherein the step of determining the homology basis includes using a spanning tree method.

63. (New) The method of claim 62, wherein the step of using the spanning tree method comprises:

- computing the fundamental domain  $D_M$  for the mesh  $M$ ;
- determining the cut graph  $G$  of the mesh  $M$ ;
- determining a spanning tree  $T$  of the cut graph  $G$ ;
- removing the spanning tree  $T$  from the cut graph  $G$  to yield a series of edges; and
- adding the edges to the spanning tree  $T$  to determine the homology basis of the mesh  $M$ .

64. (New) The method of claim 62, wherein the step of using the spanning tree method comprises:

- computing the fundamental domain  $D_M$  of the mesh  $M$ ;
- determining the cut graph  $G$  of the mesh, the cut graph  $G$  comprising a plurality of non-oriented edges attached to the cyclically ordered list of half edges comprising the boundary of the fundamental domain  $\partial D_M$ ;
- determining a spanning tree  $T$  of  $G$ ;
- removing the spanning tree  $T$  from the cut graph  $G$  to yield a series of edges, represented as  $\{e_1, e_2, \dots, e_{2g}\}$ , where  $g$  is the genus of the mesh  $M$ ;
- adding  $e_i$ , where  $i$  is an integer, to the spanning tree  $T$  to yield a single loop  $\gamma_i$ , where  $\{\gamma_1, \gamma_2, \dots, \gamma_{2g}\}$  form the homology basis of the mesh  $M$ .

65. (New) The method of claim 64, wherein the step of computing the homology basis includes using a boundary matrix method.

66. (New) The method of claim 65, wherein the boundary matrix method comprises:

- computing two boundary matrices for the mesh  $M$ ;
- forming a combinatorial Laplace matrix for the mesh; and
- computing the homology basis of the mesh  $M$ .

67. (New) The method of claim 65, wherein the boundary matrix method comprises:  
 computing two boundary matrices respectively represented as  $\partial_1, \partial_2$  for the mesh;  
 forming a combinatorial Laplace matrix  $D$  defined as  $D = \partial_2 \partial_2^T + \partial_1^T \partial_1$ , wherein  $\partial_1^T$  is the transpose of  $\partial_1$ , and wherein  $\partial_2^T$  is the transpose of  $\partial_2$ ; and  
 computing the homology basis of the mesh  $M$  as the eigenvectors of  $D$  corresponding to zero eigenvalues, represented as  $\{\gamma_1, \gamma_2, \dots, \gamma_{2g}\}$ , wherein each  $\gamma_i$ , wherein  $i$  is an integer, represents a closed loop formed by a sequence of half edges.

68. (New) The method of claim 48, wherein the step of determining the conformal structure method comprises:  
 computing a cohomology basis for the surface by determining a set of linearly independent vector fields on the surface respectively having zero curl, and respectively referred to as 1-forms.

69. (New) The method of claim 68, wherein the step of computing a cohomology basis includes using an incremental constructive method.

70. (New) The method of claim 69, wherein the step of using an incremental constructive method comprises:

- (i) computing the fundamental domain  $D_M$  of the mesh  $M$ ;
- (ii) determining the cut graph  $G$  of the mesh  $M$ ;
- (iii) determining a spanning tree  $T$  of the cut graph  $G$ ;
- (iv) removing the spanning tree  $T$  from the cut graph  $G$  to yield a series of edges, represented as  $\{e_1, e_2, \dots, e_{2g}\}$ , wherein  $g$  is the genus of the mesh  $M$ ; and
- (v) determining the 1-forms, represented as  $\omega_i$ , satisfying  $\omega_i(e_i)=1$  and  $\omega_i(e_j)=0$ , wherein  $i$  is not equal to  $j$ , and  $i = \{1, 2, \dots, 2g\}$ .

71. (New) The method of claim 69, wherein the step of using an incremental constructive method comprises:

- (i) computing the fundamental domain  $D_M$  of the mesh  $M$ ;
- (ii) determining the cut graph  $G$  of the mesh  $M$ , comprising a plurality of non-oriented edges attached to the cyclically ordered list of half edges comprising the boundary of the fundamental domain  $\partial D_M$ ;
- (iii) determining a spanning tree  $T$  of the cut graph  $G$ ;
- (iv) removing the spanning tree  $T$  from the cut graph  $G$  to yield a series of edges, represented as  $\{e_1, e_2, \dots, e_{2g}\}$ , wherein  $g$  is the genus of the mesh  $M$ ; and
- (v) for each said 1-form  $\omega_i$ , wherein  $i$  is an integer, and wherein  $i = \{1, 2, \dots, 2g\}$ :
  - (A) setting  $\omega_i(e_i)=1$  and  $\omega_i(e_j)=0$ , for any of said edges  $e_i, e_j$ , wherein  $j$  is an integer, wherein  $i$  is not equal to  $j$ ;
  - (B) for any of said edges  $e_i$  on the spanning tree  $T$ , setting the 1-form parameters  $\omega_i$  corresponding to each edge  $e_i$  to 0, such that  $\omega_i(e_i)=0$ ;
  - (C) ordering the triangular mesh faces of the fundamental domain  $D_M$  such that the fundamental domain  $D_M$  comprises  $\{f_1, f_2, \dots, f_n\}$ , wherein  $n$  is an integer;
  - (D) reversing the order of the fundamental domain  $D_M$  to comprise  $\{f_n, f_{n-1}, \dots, f_1\}$ ; and
  - (E) while the fundamental domain  $D_M$  is not empty,
    - (1) retrieving a face  $f_i$  of the fundamental domain  $D_M$ ;
    - (2) removing  $f_i$  from the fundamental domain  $D_M$ , wherein the boundary  $\partial f_i = e_0 + e_1 + e_2$ ;
    - (3) dividing  $\{e_k\}$ , wherein  $k$  is an integer, into two sets,  $\Gamma = \{e_k \in \partial f \mid e_k \in \partial D_M\}$ ,  $\Pi = \{e_k \in \partial f \mid e_k \notin \partial D_M\}$ ;
    - (4) choosing the value of  $\omega_i(e_k)$ ,  $e_k \in \Pi$  arbitrarily, such that  $\sum_{e \in \Pi} \omega_i(e_k) = -\sum_{e \in \Gamma} \omega_i(e_k)$ , and if  $\Pi$  is empty, then the right hand side is equal to zero; and
    - (5) updating the fundamental domain  $D_M$  and the boundary of the fundamental domain  $\partial D_M$  by setting  $\partial D_M = \partial D_M + \partial f$  and  $D_M = D_M - f$ .

72. (New) The method of claim 48, wherein the step of determining the conformal structure comprises:

determining a basis for the linear space of the vector fields on the surface having zero curl and zero divergence, wherein the vector fields are referred to as harmonic 1-forms of the surface, and wherein the basis is referred to as the harmonic 1-form basis of the surface.

73. (New) The method of claim 72, wherein the step of determining the harmonic 1-form basis comprises using a diffusion method.

74. (New) The method of claim 73, wherein the diffusion method comprises:

determining a cohomology basis, represented as  $\{\omega_1, \omega_2, \dots, \omega_{2g}\}$ , wherein  $g$  is the genus of the mesh  $M$ ; and

determining a function  $F_i$  on the vertices of the mesh, wherein  $i$  is an integer, wherein the Laplacian of  $(\omega_i + \delta F_i)$  equals 0, wherein  $\delta F_i$  is the gradient of  $F_i$ , and wherein  $\{\omega_1 + \delta F_1, \omega_2 + \delta F_2, \dots, \omega_{2g} + \delta F_{2g}\}$  represents the harmonic 1-form basis.

75. (New) The method of claim 73, wherein the step of determining the harmonic 1-form basis comprises using a linear algebraic method.

76. (New) The method of claim 75, wherein the step of using a linear algebraic method comprises:

(i) determining the homology basis, represented as  $\{\gamma_1, \gamma_2, \dots, \gamma_{2g}\}$ , wherein  $g$  is the genus of the mesh  $M$ ;

(ii) determining the algebraic intersection numbers  $c_i^j = -\gamma_i \bullet \gamma_j$ , wherein  $i, j = 1, 2, \dots, 2g$ ; and

(iii) for each  $i = \{1, 2, \dots, 2g\}$ , solving the linear system of three sets of equations, comprising:

$$(A) \delta \lambda_i = 0;$$

$$(B) \text{ the Laplacian of } \lambda_i = 0; \text{ and}$$

(C) the integral of  $\lambda_i$  along curve  $\gamma_j$ , represented as  $(\oint_{\gamma_j} \lambda_i)$  equals  $c_i^j$ , to yield  $\lambda_i$ ,

wherein steps (i)-(iii) yield  $\{\lambda_1, \lambda_2, \dots, \lambda_{2g}\}$  comprises the harmonic 1-form basis.

77. (New) The method of claim 48, wherein the step of determining the conformal structure comprises:

determining a basis for the linear space of the complex vector fields on the surface, wherein each of the complex vector fields respectively comprises two real vector fields, each of the real vector fields having zero curl, zero divergence, being orthogonal to each other, and being respectively referred to as harmonic 1-forms, and

wherein the complex vector fields are referred to as holomorphic 1-forms of the surface, and wherein the basis thereof is referred to as the holomorphic 1-form basis of the surface.

78. (New) The method of claim 77, wherein the step of determining the holomorphic 1-form basis comprises:

(i) if the mesh is open, then performing the following steps:

(A) closing the mesh;

(B) determining a symmetric harmonic 1-form basis of the closed mesh; and

(C) determining the conjugate harmonic 1-forms of the symmetric harmonic 1-form basis; and

(D) pairing the symmetric harmonic 1-form basis with the conjugate harmonic 1-forms to form the holomorphic 1-form basis of the open mesh; and

(ii) if the mesh is closed, then performing the following steps:

(A) determining a harmonic 1-form basis of the mesh;

(B) determining the conjugate harmonic 1-forms of the harmonic 1-form basis;

and

(C) pairing the harmonic 1-form basis with the conjugate harmonic 1-forms to form the holomorphic 1-form basis of the closed mesh.



79. (New) The method of claim 77, wherein the step of determining the holomorphic 1-form basis comprises:

(i) if the mesh is open, then performing the following steps:

(A) doubling the mesh  $M$  to form a mesh  $\bar{M}$ ;

(B) determining the harmonic 1-form basis, represented as  $\{\omega_1, \omega_2, \dots, \omega_{2g}\}$  of  $\bar{M}$ , wherein  $\omega_i$  is a harmonic 1-form for the mesh  $\bar{M}$ , and wherein  $g$  is the genus of  $\bar{M}$ ;

(C) determining the dual harmonic 1-form, represented as  $\bar{\omega}_i$  for  $i=\{1, 2, \dots, 2g\}$  such that  $\omega_i(e) = -\bar{\omega}_i(\bar{e})$ , wherein  $e$  is an edge of the mesh, wherein  $e$  and  $\bar{e}$  are dual to each other, and wherein  $i$  is an integer;

(D) assigning another harmonic 1-form  $\tau_i = (\omega_i + \bar{\omega}_i)$ , for  $i=\{1, 2, \dots, 2g\}$  and deriving a maximum linearly independent subset of  $\{\tau_1, \tau_2, \dots, \tau_{2g}\}$  denoted as  $\{\tau_1, \tau_2, \dots, \tau_k\}$ , wherein  $k$  is an integer;

(E) determining respective conjugate harmonic 1-forms of  $\tau_i$  for  $i=\{1, 2, \dots, k\}$  respectively denoted as  $\tau_i^*$ ; and

(F) determining the holomorphic 1-form basis to be  $\{\tau_1 + \sqrt{-1}\tau_1^*, \tau_2 + \sqrt{-1}\tau_2^*, \dots, \tau_k + \sqrt{-1}\tau_k^*\}$ ; and

(ii) alternatively, if the mesh is closed, then performing the following steps:

(A) determining the harmonic 1-form basis, represented as  $\{\omega_1, \omega_2, \dots, \omega_{2g}\}$ , wherein  $\omega_i$  is a harmonic 1-form for the mesh, and  $g$  is the genus of the mesh;

(B) determining conjugate harmonic 1-forms of  $\omega_i$  denoted as  $\omega_i^*$ , wherein  $i$  is an integer; and

(C) determining the holomorphic 1-form basis to be  $\{\omega_1 + \sqrt{-1}\omega_1^*, \omega_2 + \sqrt{-1}\omega_2^*, \dots, \omega_{2g} + \sqrt{-1}\omega_{2g}^*\}$ .

80. (New) The method of claim 48, wherein the step of determining the conformal structure further comprises determining the period matrix of the mesh by integrating the holomorphic 1-form basis of the mesh along a homology basis of the mesh.

81. (New) The method of claim 80, wherein the step of determining the period matrix further includes:

determining the homology basis, represented as  $\{\gamma_1, \gamma_2, \dots, \gamma_{2g}\}$ ;

determining the holomorphic 1-form basis to be  $\{\omega_1 + \sqrt{-1}\omega_1^*, \omega_2 + \sqrt{-1}\omega_2^*, \dots, \omega_{2g} + \sqrt{-1}\omega_{2g}^*\}$ ;

determining the harmonic 1-form basis, represented as  $\{\omega_1, \omega_2, \dots, \omega_{2g}\}$ , wherein  $\omega_i$  is a harmonic 1-form for the mesh, wherein  $g$  is the genus of the mesh, and wherein  $i$  is an integer;

determining the elements of a matrix  $C$  as  $c_{ij} = \oint_{\gamma_i} \omega_j$ , said matrix  $C$  also being

represented as  $\langle \gamma_i, \omega_j \rangle$ ;

determining the elements of a matrix  $S$  as  $s_{ij} = \oint_{\gamma_i} \omega_j^*$ , said matrix  $S$  also being

represented as  $\langle \gamma_i, \omega_j^* \rangle$ ; and

determining the matrix  $R$  as the solution of  $CR = S$ , wherein  $R$  satisfies  $R^2 = -I$ , wherein  $I$  is the identity matrix, and wherein  $R$  is the period matrix.

82. (New) The method of claim 48, wherein the surface is a closed, genus 0 surface, and wherein the canonical domain is a sphere.

83. (New) The method of claim 82, wherein the step of using the conformal structure to conformally map the surface comprises using a heat diffusion method.

84. (New) The method of claim 83, wherein heat diffusion method comprises:

(i) determining the Gauss map  $\phi$  which maps the mesh  $M$  to the sphere  $S^2$ ;

(ii) determining the Laplacian at each vertex  $u$  of the mesh  $M$ , denoted as  $\Delta\phi(u)$ ;

(iii) projecting  $\Delta\phi(u)$  to the tangent space of the image  $\phi(u)$  of the vertex  $u$ , wherein  $\phi(u)$  is on the sphere  $S^2$ ;

(iv) updating  $\phi(u)$  along the negative direction of the projected  $\Delta\phi(u)$ ;

(v) determining the center of mass  $mc(\phi)$  of  $\phi(u)$ ;

(vi) shifting the center of mass  $mc(\phi)$  to the center of  $S^2$ ;

(vii) renormalizing  $\phi(u)$  to be on  $S^2$ ; and

(viii) repeating steps (ii)-(vii) for all remaining vertices  $u$  of the mesh  $M$ , until the projected Laplacian of all vertices equals zero.

85. (New) The method of claim 48, wherein the surface is an open, genus zero surface with one boundary, referred to as a topological disk, and wherein the canonical parameter domain is a canonical planar disk.

86. (New) The method of claim 85, wherein the step of using the conformal structure to conformally map the surface comprises:

doubling the mesh  $M$  to form a second mesh  $\bar{M}$ ;

conformally mapping  $\bar{M}$  to a sphere  $S^2$ ; and

conformally mapping a hemisphere of the sphere  $S^2$  to the canonical planar disk.

87. (New) The method of claim 77, wherein the step of using the conformal structure to conformally map the surface comprises:

integrating the holomorphic 1-form of the surface to map the surface onto the canonical parameter domain.

88. (New) The method of claim 87, wherein the step of integrating the holomorphic 1-form of the surface to map the surface onto the canonical parameter domain comprises:

a) specifying  $\{\omega_i + \sqrt{-1}\omega_i^*\}$  as the holomorphic 1-form basis, wherein  $\tau_i$  represents the harmonic 1-form of the mesh  $M$ ,  $\tau_i^*$  represents the conjugate harmonic 1-form of the mesh  $M$ , and wherein  $i$  is an integer;

b) determining a partition represented as  $\{U_i\}$ , such that each  $U_i$  is a topological disk, and the mesh  $M$  is included in union of all  $U_i$ ;

- c) for each  $U_i$ , selecting a holomorphic 1-form basis  $\omega_j + \sqrt{-1}\omega_j^*$ ;
- d) integrating the holomorphic 1-form  $\omega_j + \sqrt{-1}\omega_j^*$  on  $U_i$ , denoting said mapping as  $z_i$ , and in the event that there are zero points, subdividing  $U_i$  and repeating said integration; and
- e) determining the conformal structure to be  $\{(U_i, z_i)\}$ .

89. (New) The method of claim 88, wherein step d) comprises:  
selecting a root vertex represented as  $v_0$ ;  
for all vertices  $u$ , finding a path from  $v_0$  to  $u$ ;  
computing the integration of  $\phi(u) = \langle \omega, \gamma_u \rangle$ , wherein  $\omega$  is the holomorphic 1-form,  $\gamma_u$  is the path from  $v_0$  to vertex  $u$ , and  $\phi(u)$  is the parameterization of the surface patch  $U_i$ ,  
wherein  $(\phi(u), U_i)$  is a parameter chart, and wherein the union of all said parameter charts  $(\phi(u), U_i)$  comprise the conformal structure.

90. (New) The method of claim 48, wherein the surface is a closed, genus one surface, and wherein the canonical domain is a Euclidian plane.

91. (New) The method of claim 48, wherein the surface is a closed surface having genus greater than one, referred to as a high genus surface, and wherein the canonical domain is a Euclidean plane.

92. (New) The method of claim 48, wherein the surface is a closed surface having genus greater than one, referred to as a high genus surface, and wherein the canonical domain is a hyperbolic plane.

93. (New) The method of claim 48, wherein the surface is an open, genus zero surface with two boundaries, referred to as an annulus, and wherein the canonical domain is a Euclidean plane.

94. (New) The method of claim 48, wherein the surface is an open surface of any genus, wherein the surface is not any one of a topological disk or an annulus, and wherein the canonical domain is a Euclidean plane.

95. (New) The method of claim 48, wherein the surface is an open surface of any genus, wherein the surface is not any one of a topological disk or an annulus, and wherein the canonical domain is a hyperbolic plane.

96. (New) The method of claim 48, wherein the surface conformally mapped to the canonical parameter domain is used for a graphics application.

97. (New) The method of claim 96, wherein the graphics application comprises an animation application.

98. (New) The method of claim 97, comprising respectively animating the mesh representation  $M$  of the surface from a first mesh representation,  $M_1$ , to a second mesh representation,  $M_2$ , the method comprising the steps of:

- (i) removing at least one feature point common to both  $M_1$  and  $M_2$ ;
- (ii) doubling each of  $M_1$  and  $M_2$ , respectively;
- (iii) conformally mapping  $M_1$  and  $M_2$  to respective first and second canonical surfaces to respectively form first and second canonical mapped surfaces;
- (iv) determining a holomorphic 1-form of said first and second mapped surfaces;
- (v) determining a cohomology basis of said first and second mapped surfaces;
- (vi) locating respective first and second zero points on said first mesh representation  $M_1$  and said second mesh representation  $M_2$ ;
- (vii) determining respective first and second gradients of said first and second mapped surfaces at said first and second zero points respectively;
- (viii) decomposing the first and second mesh representations,  $M_1$  and  $M_2$ , respectively, into respective first and second sets of canonical patches using integration lines along said first and second gradients and through said first and second zero points, respectively;

(ix) conformally mapping said first and second sets of patches onto first and second rectangles, respectively;

(x) matching said first and second mapped patches respectively on said first and second rectangles to form a map therebetween;

(xi) selecting at least one control point on each of said first and second mapped patches on said first and second rectangles, respectively; and

(xii) using a BSpline to generate a smooth transition of said mapping to said first mesh representation  $M_1$  and said second mesh representation  $M_2$ .

99. (New) The method of claim 96, wherein the graphics application comprises a texture mapping application.

100. (New) The method of claim 99, comprising texture mapping on the mesh representation  $M$  representative of the surface, the method comprising the steps of:

removing at least one feature point on the mesh  $M$ ;

doubling the mesh  $M$ ;

conformally mapping the mesh  $M$  to a canonical surface to form a mapped surface;

determining a holomorphic 1-form for said mapped surface;

determining a cohomology basis for said mapped surface;

locating one or more zero points on said mapped surface;

determining a gradient of said mapped surface;

decomposing said mesh representation,  $M$ , into at least two canonical patches using respective integration lines along said gradient and through one or more of said zero points, respectively;

conformally mapping said at least two canonical patches onto at least two rectangles;

growing said at least two rectangles until respective boundaries between said at least two rectangles meet;

fixing the boundaries between said at least two rectangles; and

solving the Dirichlet problem for the respective uncovered regions of said at least two rectangles.

101. (New) The method of claim 96, wherein the graphics application comprises a geometric modeling application.

102. (New) The method of claim 101, wherein the geometric modeling application comprises converting the mesh M to a spline for a spline conversion application.

103. (New) The method of claim 102, comprising:  
generating a conformal representation of the mesh M;  
determining the gradient of the mesh M;  
determining the zero points of the mesh M;  
decomposing the mesh M into canonical patches using integration lines along the gradient and through one or more of said zero points;  
conformally mapping the canonical patches onto a respective rectangle;  
constructing a tensor product spline surface on the surface of the rectangle; and  
matching the control points on the boundary of the rectangle to make the parameterization globally smooth.

104. (New) The method of claim 101, wherein the geometric modeling application comprises converting the mesh from an irregular format to a regular format, said application being referred to as a remeshing application.

105. (New) The method of claim 96, wherein the graphics application comprises a compression application.

106. (New) The method of claim 105, wherein the compression application comprises compressing the mesh M, the method comprising:  
conformally mapping the mesh M to a canonical shape;  
representing the surface position vector on the canonical shape as a vector valued function;

finding the eigen functions of the Laplacian of the mesh M;  
decomposing the vector valued function using the eigen functions of the Laplacian;  
removing the high frequency components of the decomposed vector valued function; and  
storing the low frequency components of the decomposed vector valued function as a  
compressed image of the mesh representation, M, of the surface.

107. (New) The method of claim 105, wherein the compression application comprises compressing the mesh M, the method comprising:

conformally mapping the mesh M to a canonical parameter domain forming a mapped surface;

determining a conformal parameterization of said mapped surface;

determining a level set of Gaussian curvature and mean curvature for said mapped surface, wherein said level set of Gaussian curvature and mean curvature is a function of said conformal parameterization; and

storing said conformal parameterization of the mesh M and said level set corresponding to the mesh M.

108. (New) The method of claim 48, wherein the surface conformally mapped to the canonical parameter domain is used for any one of:

a geometric database and search application; and

a geometric shape analysis application.

109. (New) The method of claim 108, further comprising performing a surface comparison.

110. (New) The method of claim 109, wherein the surface comparison comprises performing a verification analysis between the surface and a second surface.

111. (New) The method of claim 110, wherein the step of performing a verification analysis comprises:



performing steps (i) and (ii) of claim 1 for the second surface; and  
verifying the identity of the surface by comparing the surface to the second surface.

112. (New) The method of claim 111, wherein said surface, referred to as a first surface, and said second surface respectively belong to the same class.

113. (New) The method of claim 111 comprising determining whether a match exists between said surface, referred to as a first surface, and said second surface, the method comprising:

conformally mapping both the mesh representation corresponding to said first surface, referred to as a first mesh representation, and a second mesh representation corresponding to said second surface, to a first canonical parameter domain to respectively form first and second mapped surfaces;

determining respective first and second conformal parameterizations of said first and second mapped surfaces, respectively;

determining respective first and second level sets of Gaussian curvature and mean curvature for said first and second mapped surfaces, respectively, wherein said first and second level sets of Gaussian curvature and mean curvature are functions of said first and said second conformal parameterizations, respectively; and

comparing said first and second level sets of Gaussian curvature and mean curvature, and in the event said comparison exceeds a predetermined threshold, determining a match to exist between said first and said second surface, and otherwise determining a mismatch to exist between said first and said second surface.

114. (New) The method of claim 111, comprising determining whether a match exists between said surface, referred to as a first surface, and said second surface, the method comprising

removing substantially all feature points from the mesh representation M;

selecting an arbitrary point on the surface of the mesh representation M;

selecting an arbitrary orbit along the surface of the mesh representation M for the arbitrary point to follow;

moving the arbitrary point along the arbitrary orbit;

removing the arbitrary point from the mesh representation M at least one point along the arbitrary orbit;

doubling the mesh representation M having all feature points removed and the arbitrary point removed;

determining at least one period matrix R for the mesh representation having the arbitrary point removed;

comparing said period matrix R with at least one standard period matrix corresponding to a standard surface; and

in the event that said period matrix, R and said standard period matrix are within a predetermined factor of one another, providing indicia that the surface represented by the mesh representation is recognized as being substantially similar to the at least one standard surface.

115. (New) The method of claim 111, wherein the canonical parameter domain is used for the geometric database and search application, and wherein the surface comprises at least a portion of a human face.

116. (New) The method of claim 111, wherein the canonical parameter domain is used for the geometric search analysis application comprising a medical imaging application, and wherein the surface comprises at least a portion of a human organ.

117. (New) The method of claim 116, wherein the canonical parameter domain is a sphere, and wherein the human organ is a brain.

118. (New) The method of claim 116, wherein the canonical parameter domain is a planar rectangle, and wherein the human organ is a colon.

119. (New) The method of claim 108, wherein the geometric database and search application comprises performing a surface classification.

120. (New) The method of claim 108, wherein the geometric database and search application further comprises performing a surface comparison.

121. (New) The method of claim 120, wherein the surface comparison comprises performing an identification analysis between the surface and a plurality of second surfaces.

122. (New) The method of claim 121, wherein the step of performing an identification analysis comprises:

performing steps (i) and (ii) of claim 1 for each of the second surfaces; and  
identifying the surface by comparing the surface to the plurality of second surfaces.

123. (New) The method of claim 122, wherein said surface, referred to as a first surface, and said second surfaces respectively belong to different classes.

124. (New) The method of claim 123, wherein the period matrix of each of said surfaces is calculated and used to determine which of said surfaces belong to the same classes.

125. (New) The method of claim 124, further comprising performing the method of claim 109 for any two of said surfaces belonging to the same class.

126. (New) The method of claim 124, further comprising performing the method of claim 110 for any two of said surfaces belonging to the same class.

127. (New) The method of claim 124, further comprising performing the method of claim 111 for any two of said surfaces belonging to the same class.

128. (New) The method of claim 124, further comprising performing the method of claim 113 for any two of said surfaces belonging to the same class.

129. (New) The method of claim 108, wherein the surface and the plurality of second surfaces comprise any one of:

at least a portion of a human face for a face identification application; and

at least a portion of a three dimensional object stored as a data file in a shape retrieval application, comprising at least a portion of any one of:

a data file for storing a manufactured mechanical part;

a data file accessible from an Internet web page;

a data file used for video gaming;

a data file used for a motion picture; and

a data file used for a biometric application.

130. (New) The method of claim 109, wherein the surface comparison comprises using a geometric search engine to perform a search analysis of a geometric database.

131. (New) A method of volumetric harmonic mapping of a mesh representation M of a 3-dimensional manifold, the method comprising the steps of:

computing the harmonic energy of a mapping  $f : M \rightarrow R^3$ , wherein the harmonic energy is given by  $E(f) = \sum_{[u,v] \in M} k_{uv} \|f(u) - f(v)\|$ , wherein u is a first vertex, wherein v is a second vertex,

wherein  $f()$  is the harmonic mapping of u, v, and wherein  $k_{uv} = \frac{\lambda}{48} \sum_{\theta} \cot(\theta)$ , wherein  $\theta$  represent dihedral angles opposite to the given edges connecting u, v, and wherein  $\lambda$  is the length of a said given edge; and

minimizing the harmonic energy using the conjugate gradient method to obtain the harmonic mapping, f.

132. (New) The method of claim 131, wherein the representation  $M$  of the 3-dimensional manifold is a magnetic resonance image of a body area of interest, and wherein the mapping  $f: M \rightarrow R^3$  is a map onto a canonical sphere of the body area of interest.

133. (New) The method of claim 132, further comprising using said map onto said canonical sphere of said body area of interest for surgical planning.